Aerodynamics in a Hydrogen Atmosphere: Supersonic Tube Vehicle

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Abstract

Operation of a vehicle in a hydrogen atmosphere, because of its low density, would increase sonic speed and dramatically decrease drag relative to air. A hydrogen atmosphere requires that the vehicle operate in a hydrogen-filled tube or pipeline; hydrogen pressure is slightly above outside air pressure, and the tube serves as a phase separator. A cross between a train and airplane, the vehicle utilizes fuelcell-powered propfan propulsion and gas-bearing levitation on a guideway within the tube. Fuel is breathed from the tube. Because a tube has an inside and an outside, the vehicle has two Mach numbers, one with respect to hydrogen inside the tube and one with respect to air outside. While fundamental aerodynamic concepts apply to hydrogen as readily as to air, aerodynamics in a tube is the principal challenge. In this paper, we will focus on the tube-vehicle phenomenon of accelerated gas flow, termed *gap flow*, through the gap formed by the vehicle outer surface and the tube inner surface. I derive equations for the drag due to gap flow and the power required to overcome it. The equations for gap flow, gap drag, and gap power have allowed several design advances in the supersonic tube vehicle concept.

Nomenclature

A_g = cross-sectional area of the gap (m^2)
A_t = cross-sectional area of the tube (m^2)
C_f = vehicle drag coefficient corresponding to frontal area
$C_{f\infty}$ = vehicle drag coefficient corresponding to frontal area in an infinite-diameter tube
D_t = drag on the inner surface of the tube
D_v = drag on vehicle surface in a finite-diameter tube
$D_{v\infty}$ = drag on vehicle surface in an infinite-diameter tube
δ_t = tube inside diameter (m)
δ_v = vehicle outer diameter as a function of distance along its centerline (m)
ΔV = increment of gas velocity in the gap between vehicle and tube (m/s)
L = length of the vehicle frontal surface (m)
M = entire length of a trainset, including two locomotives and intervening passenger cars (m)
P_t = required power to overcome drag on inner surface of the tube (MW)
P_v = required vehicle power in the gap of a finite-diameter tube (MW)
$P_{v\infty}$ = required vehicle power in an infinite-diameter tube (MW)
q = ratio of tube diameter to vehicle diameter
ρ = gas density in a finite-diameter tube (kg/m^3)
ρ_g = gas density in the gap (kg/m^3)
ρ_t = gas density far ahead of the vehicle; same as ρ_∞ (kg/m^3)
ρ_∞ = gas density in an infinite-diameter tube (kg/m^3)
S_f = frontal area of the vehicle (m^2)
S_t = area of the inner surface of the tube
S_v = entire surface area of the vehicle
V_g = Gas velocity in the gap (m/s)
V_∞ = vehicle velocity with respect to the tube or to the tube atmosphere in a tube of infinite diameter (m/s)
x = distance along vehicle centerline, with $x = 0$ corresponding to the vehicle tip (m)
x^* = a maximum point for a function of x

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I. Introduction

The supersonic tube vehicle is a new concept in high-speed transport.[1,2] Operation of a vehicle in a hydrogen atmosphere, because of its low density, would increase sonic speed and dramatically decrease drag relative to air. A hydrogen atmosphere requires that the vehicle operate in a hydrogen-filled tube or pipeline. To prevent leakage of air into the tube, hydrogen pressure is slightly above outside air pressure, and the tube serves as a phase separator.

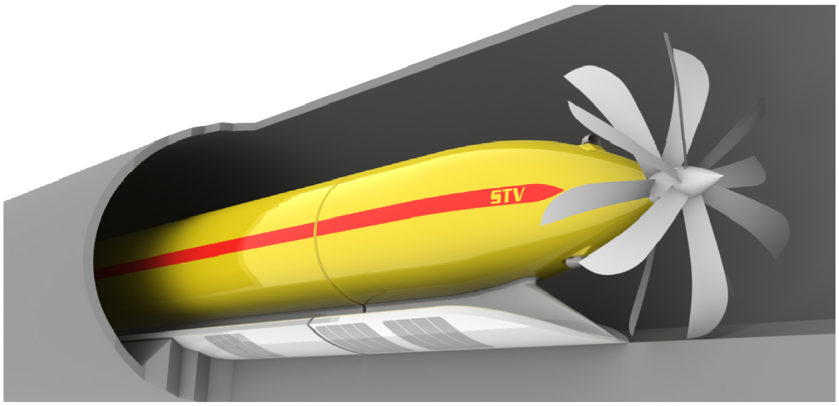

Figure 1. Supersonic tube vehicle in its hydrogen tube. Cutaway model shows front locomotive, first passenger car, contra-rotating propfan propulsion, and levitation on aerostatic gas bearings. Fuselage outer diameter is 2.69 m, propeller diameter is 4.5 m, and tube inside diameter is 6.4 m. Tube hydrogen pressure is slightly above outside air pressure.

Because a tube has an inside and outside, the vehicle has two Mach numbers, one with respect to hydrogen inside the tube and one with respect to air outside. Based on the ratio of the speed of sound in the two phases, a speed of Mach 2.8 with respect to air outside the tube corresponds to only Mach 0.74 inside. Thus, the vehicle can be strongly supersonic outside while remaining subsonic inside.

Because energy consumption due to parasitic drag is a function of the first power of gas density, the hydrogen atmosphere would reduce energy consumption at a given speed by a factor of 15 relative to air. Actually, the energy consumption relative to an airplane may be lower by nearly a factor of 30: Besides lower parasitic drag, the supersonic tube vehicle does not experience induced drag, the source of half the energy consumption of an airplane minimized for total drag.

For these reasons, the supersonic tube vehicle is capable in theory of cruising at Mach 2.8 and concurrently consuming less than half the energy per passenger of a Boeing 747-400 at a cruise speed of Mach 0.81 [Ref. 2].

This paper reports my preliminary studies on the aerodynamics of a vehicle operating in a hydrogen atmosphere within a tube.[3] Fundamental aerodynamic concepts such as the perfect-gas law, continuity equation, and Bernoulli's equation are functions of gas density – not identity of the gas – and therefore apply to hydrogen as readily as to air. However, to operate a vehicle in a hydrogen atmosphere requires that it operate within a tube, and aerodynamics in a tube is the principal challenge. We will focus here on the tube-vehicle phenomenon of accelerated gas flow, termed *gap flow*, through the gap formed by the vehicle outer surface and the tube inner surface. I will derive equations for the drag due to gap flow and the power required to overcome it.

II. Background

The proposed supersonic tube vehicle has characteristics of both a train and an airplane. It consists of a string of multi-articulated passenger cars intervening two locomotives, one pulling and one pushing, all running on a guideway within the tube. A trainset levitates above the guideway on aerostatic gas bearings or magnetic fields. The locomotives are propelled by contra-rotating propfans, which are powered by onboard hydrogen-oxygen fuel cells. Hydrogen fuel is breathed from the tube, liquid-oxygen (LOX) oxidant is carried onboard, and product water is collected and stored onboard until the end of a run. Breathing fuel from the tube solves the challenging problem of hydrogen storage for long-range hydrogen fuel-cell vehicles.

Speed of the supersonic tube vehicle as thus conceived is limited by shock-formation at the propeller-blade tips, and the practical maximum cruise speed was estimated as Mach 0.74 by aerodynamic extrapolation of the cruise speed of the turbopropfan Antonov An-70.[2] Because the Mach number for onset of shock waves in hydrogen should be the same as that in air,[3] the maximum cruise speed of the vehicle should be larger than that of the An-70 by the ratio of the speed of sound in hydrogen at 101 kPa to its speed in air at 10 050 m altitude, the cruise altitude of the An-70.[4] Thus, the maximum cruise speed was estimated as 3500 km/h (Mach 0.74) in hydrogen, which corresponds to Mach 2.8 in air at sea level outside the tube.

Required propulsion power P of the supersonic tube vehicle was estimated as the power to propel the Bombardier Q400 fuselage, as a model, in hydrogen. From the known power required by the entire Q400 airplane in air, its value in hydrogen was calculated, followed by mathematically clipping off its wings and other appendages to give the bare fuselage operating in a hydrogen atmosphere.[1,5] The method estimated an upper bound of $P \leq 2$ MW at Mach 0.32 in hydrogen (Mach 1.2 with respect to air); for the purposes of calculation, the value $P \cong 2.0$ MW was used. From this baseline calculation, the power and energy at higher speeds were calculated from the fact that power increases as the cube of speed and energy as the square. Refinement of this estimate by using the results of the work at hand reduces the uncertainty of the upper bound, but I will show below that $P \leq 2$ MW is still an upper bound at Mach 0.32 in hydrogen.

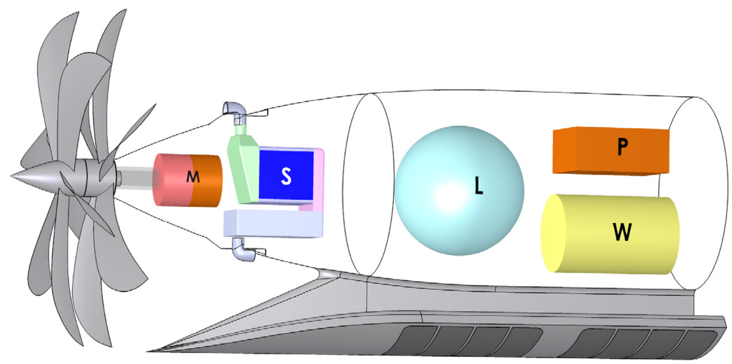

Fig 2. X-ray view of a supersonic tube vehicle locomotive. This CAD model illustrates feasibility of the following for supersonic transit: (a) Fuelcell stacks of the required power can be packaged within the vehicle; (b) adequate LOX can be carried onboard to allow nonstop transcontinental operation; and (c) water produced during the run can be stored onboard. The components shown are M = propulsion motor, S = fuel-cell stacks, L = LOX system, P = power electronics, and W = water holding tank. Components S, L, and W are to the scale of the fuselage maximum diameter of 2.69 m.

Comparison of the normalized energy consumption of the supersonic tube vehicle with Boeing 747 and Bombardier Q400 aircraft gave the result, among others, that the supersonic tube vehicle at Mach 2.8 consumes less than half the energy per passenger per kilometer of the 747 at Mach 0.81. Empirical energy consumption data for the airplanes utilized more than 28 thousand data points.[2]

Some aspects of feasibility of the concept have been analyzed. As shown in Figure 2, these include calculations showing that the vehicle is able to carry sufficient LOX and water to make a nonstop transcontinental supersonic run of 3960 km.[1]

The cited papers [1,2,3] comprise the first three papers of a planned series, with main title "Hydrogen Tube Vehicle for Supersonic Transport," with the objective of analyzing the scientific foundations and feasibility of the supersonic tube vehicle. Topics to be covered by the series include (a) the concept, (b) comparison of speed and energy with other transport modes, (c) aerodynamics in hydrogen, (d) levitation, (e) hazard analysis, and (f) total cost – the sum of infrastructure, operating, maintenance, and social costs. The first paper of the series,[1] though not numbered in its title, focused on the vehicle concept and its validation but also addressed in a preliminary manner engineering-design issues such as a seating arrangement in a passenger car, engineering challenges facing gas-bearing levitation, and methods of removing air or water inadvertently introduced into the tube hydrogen.

III. Gap Flow and Drag

New aerodynamic phenomena occur when a high-speed vehicle operates within a tube. As the vehicle advances, the tube atmosphere must accelerate through the gap formed between the vehicle and the inner surface of the tube. This accelerated flow, termed *gap flow*, gives rise to three aerodynamic effects: (a) Gas velocity over the vehicle surface exceeds that experienced by a free-stream vehicle, (b) gas-tube relative velocity in the gap, unlike far ahead of or behind the vehicle, is not zero, and (c) these velocity increments give rise to increased drag on the surface of the vehicle and a new type of drag on the inner surface of the tube termed *tube drag*. Collectively, these new sources of drag are termed *gap drag*.

Figure 3 describes the model of gap flow. The vehicle consists of a long, slender cylinder with nose cones at either end. The model ignores the gas-bearing or magnetic levitation structure. At point J_t far ahead of the advancing vehicle, gas velocity is stationary with respect to the tube. Gas velocity with respect to the vehicle at J_t is denoted V_∞, a reference velocity corresponding to the relative velocity of vehicle and gas in a tube of infinite diameter. V_∞ is analogous to the free-stream velocity of an airplane. A_t is the fixed cross-sectional area of the cylindrical tube at point J_t. I assume inviscid, one-dimensional flow, and let x be distance along the vehicle centerline, with $x = 0$ corresponding to the advancing tip of the vehicle. Vehicle diameter and consequent gap width vary as x increases in the nose-cone region and are then constant in the cylindrical region. The point labeled J_g represents any point in the gap flowfield; I assume the flowfield is uniform across all streamlines at any x. Let $V_g(x)$ be the gap flow velocity at point x, and $A_g(x)$ be the corresponding cross-sectional area of the annular-shaped space that comprises the gap. The variable flow velocity over the vehicle surface is given by

$$V_g(x) = V_\infty + \Delta V(x) \tag{1}$$

where $\Delta V(x)$ is the increment in gas velocity, relative to V_∞, due to acceleration through the gap.

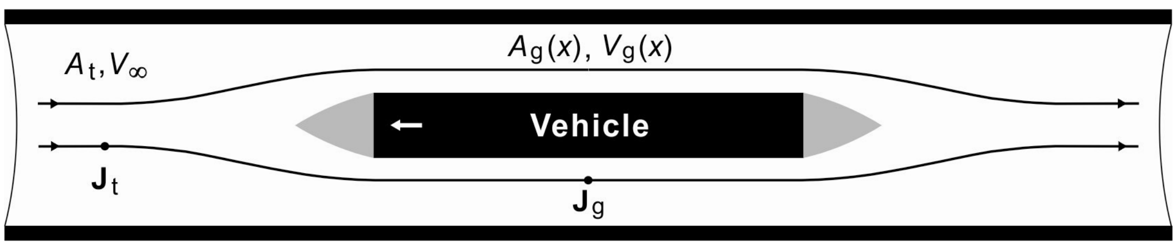

Figure 3. Model of gap flow. The vehicle is moving right-to-left in the hydrogen tube. Point J_t on a streamline is far ahead of the advancing vehicle and J_g is on the same streamline in the gap. Distance x describes the one-dimensional flow through the gap.

Acceleration of gas through the gap is governed by the continuity equation, a variation of the principle of conservation of mass, and for each value of x

$$\rho_t A_t V_\infty = \rho_g A_g(x) V_g(x) \tag{2}$$

where ρ_t and ρ_g are, respectively, the gas density at point J_t and J_g. Assume that the gas is incompressible and therefore $\rho_t = \rho_g$. Hence, we have

$$A_t V_\infty = A_g(x) V_g(x) \tag{3}$$

for the relation governing flow velocity, relative to the vehicle, through the gap.

Is the assumption of incompressibility valid? Although superficially it might seem that the tube is a duct[6] and will experience high-speed flow, this is not the case. Although high-speed flow occurs relative to the surface of the vehicle, at points either far ahead of or behind the vehicle, the relative velocity of gas and tube is zero. No flow occurs in the hydrogen tube itself except locally in the gap, and the gap flow velocity is $\Delta V(x)$. As I will show below, the maximum value of $\Delta V(x)$ is in the low-subsonic range, at most Mach 0.16 in a tube of reasonable diameter. Hence, I propose that the assumption of incompressibility is valid.†

We obtain Vg from Eq. (3) and the formulas for the area of a circle and an annulus:

$$V_g(x)/V_\infty = A_t / A_g(x) = \delta_t^2 / [\delta_t^2 - \delta_v(x)^2] \quad (4)$$

where δ_t is the fixed diameter of the tube and $\delta_v(x)$ is the diameter of the circular cross-section of the vehicle at point x. Defining the ratio of tube diameter to variable vehicle diameter as

$$q(x) = \delta_t / \delta_v(x) \quad (5)$$

we have from Eq. (5)

$$V_g(x) = V_\infty q(x)^2 / (q(x)^2 - 1), \; q(x) > 1 \quad (6)$$

as the final result for gap flow velocity over the vehicle.

We obtain ΔV, also a function of x, from the definition given by Eq. (1), namely,

$$\Delta V(x) = V_g(x) - V_\infty \quad (7)$$

which gives the flow velocity over the tube. Substituting Eq. (6) into (7) and simplifying gives

$$\Delta V(x) = V_\infty / [q(x)^2 - 1], \; q(x) > 1 \quad (8)$$

which is the desired equation for gap flow velocity over the inner surface of the tube. Dividing Eq. (8) into (6) gives the alternative result

$$\Delta V(x) = V_g(x) / q(x)^2 \quad (9)$$

These results – equations (6), (8), and (9) – apply to any gas serving as a tube atmosphere as long as the assumptions of the model are satisfied.

From the results above on gap flow, I will derive equations for the *vehicle drag ratio* and *vehicle power ratio*, the ratios of vehicle drag and power, respectively, in a finite-diameter tube to their values in an infinite-diameter tube. I will also present an equation for the distinct quantity describing tube drag termed *tube power ratio*. Tube drag results from flow, with velocity $\Delta V(x)$, over the inner surface of the tube when gas accelerates through the gap. The tube power ratio is the ratio of tube power in a tube of finite diameter to vehicle drag in the same tube.

With the abovementioned vehicle and tube power ratios, we will determine the power penalty arising from gap flow in a finite-diameter tube.

The ratio of vehicle drag in a tube of finite diameter to one of infinite diameter, the vehicle drag ratio, is given by the ratio of parasitic drags

† To the extent that compressibility does occur, Eq. (3) describes an upper bound on $V_g(x)$ because compressibility would increase the gas density in the gap: By solving Eq. (2) for $V_g(x)$, we have $V_g(x) = (\rho_t / \rho_g)\, A_t\, V_\infty / A_g(x)$ and thus $V_g(x)$ decreases when $\rho_g > \rho_t$.

$$D_{\mathrm{v}} / D_{\mathrm{v}\infty} = {}^{1}/_{2}\, C_{\mathrm{f}} \rho S_{\mathrm{f}} [\int_{0}^{L} V_{\mathrm{g}}(x) \mathrm{d}x\, L^{-1}]^{2} / ({}^{1}/_{2}\, C_{\mathrm{f}\,\infty} \rho_{\infty} S_{\mathrm{f}}\, V_{\infty}{}^{2}) \quad (10)$$

where D_{v} is vehicle drag in a finite-diameter tube, $D_{\mathrm{v}\infty}$ is vehicle drag in an infinite-diameter tube, x is distance along the centerline of the vehicle, L is the length along the centerline of the frontal surface, C_{f} is the vehicle drag coefficient, ρ is gas density in the finite-diameter tube, S_{f} is the frontal (cross-sectional) area of the vehicle, Vg(x) is gas velocity over the vehicle in the finite-diameter tube at point x, $C_{\mathrm{f}\infty}$ is the vehicle drag coefficient in the infinite-diameter tube, ρ_{∞} is gas density in an infinite-diameter tube,, and V_{∞} is gas velocity relative to the vehicle in an infinite-diameter tube. The definite integral divided by L, the length of the interval [0, L], is the mean value of function V_{g}, which is defined in Eq. (6).

Equation (10) can be simplified by canceling equal, constant factors. Coefficients C_{f} and $C_{\mathrm{f}\infty}$ are equal because the drag coefficient is a function of the nature of the surface and is unchanged by the tube diameter. Density ρ and ρ_{∞} are equal because the density is the same everywhere by the incompressibility assumption. Thus, Eq. (10) becomes

$$D_{v} / D_{v\infty} = [\int_{0}^{L} V_{g}(x) dx\, L^{-1} / V_{\infty}]^{2} \quad (11)$$

which is a simpler, equivalent definition of vehicle drag ratio when incompressibility applies. As shown by Eq. (6), V_{g}(x) is a function of V_{∞}, and thus V_{∞} is in both numerator and denominator of Eq. (11). Vehicle drag ratio is therefore independent of vehicle velocity V_{∞}.

I express the vehicle drag ratio in terms of the ratio $q(x)$ as given by Eq. (5). As before, function δ_{v} (generally a table) gives the diameter of the fuselage at each point x along the centerline. Substituting Eq. (6) into Eq. (11), and canceling the factor V_{∞}, gives

$$D_{v} / D_{v\infty} = [\int_{0}^{L} q(x)^{2} L^{-1} / (q(x)^{2} - 1)\, \mathrm{d}x]^{2} \quad (12)$$

as the final equation for vehicle drag ratio.

Because power $P = DV$, where D is drag and V is velocity, Eq. (11) shows that the vehicle power ratio is simply related to the drag ratio of Eq. (12) and is given in final form as

$$P_{v} / P_{v\infty} = [\int_{0}^{L} q(x)^{2} L^{-1} / (q(x)^{2} - 1)\, \mathrm{d}x]^{3} \quad (13)$$

where P_{v} is the vehicle power to overcome gap drag in a finite diameter tube and $P_{\mathrm{v}\infty}$ is vehicle power in an infinite-diameter tube.

Values for the vehicle drag and power ratios were computed by numerical integration. Using vehicle diameter data δ_{v} taken from the CAD model, tube diameter δ_{t} = 6.4 m, and length of the leading nose cone L = 6.70 m, Eqs. (12) and (13) respectively give the ratios as

$$D_{v} / D_{v\infty} = 1.22 \quad (14)$$

$$P_{v} / P_{v\infty} = 1.34 \quad (15)$$

Thus, gap flow increases vehicle drag by 22 % and required vehicle power by 34 %.

Because it is lengthy, I will omit the derivation for tube drag and report only the result for the tube power ratio. The tube power ratio is derived [3] as the upper bound

$$P_t / P_v \leq (S_t / S_v)[\int_0^M \Delta V(x)^2 \, dx \, M^{-1} / \int_0^L V_g(x) \, dx \, L^{-1}]^3 \tag{16}$$

where P_t is power to overcome tube drag in a finite diameter tube, P_v is vehicle power in the same tube, S_t is the surface area of the inside of the tube that surrounds the vehicle, S_v is the entire surface area of the vehicle, M is the entire length of the vehicle, a trainset, $\Delta V(x)$ is the increment in gas velocity over the inner surface of a finite-diameter tube at point x, and where V_g and L have the same definitions as in Eq. (10).

This upper bound for tube power ratio was evaluated by numerical integration. For a trainset consisting of two locomotives and four intervening passenger cars, M = 106.6 m. With tube diameter δ_t = 6.4 m, tube inner surface area S_t = 2144 m^2; with trainset maximum diameter $\delta_v(x^*)$ = 2.69 m for maximum point x^*, the surface area of two locomotives taken from the CAD model and four passenger cars is S_v = 859.8 m^2. Vehicle diameter data δ_v is taken from the CAD model, and length of the frontal surface, as before, is L = 6.70 m. Numerical integration of Eq. (16) gives

$$P_t / P_v \leq 0.0144 \tag{17}$$

as an upper bound on tube power ratio. Thus, gap flow over the tube inner surface increases the required vehicle power in the tube by no more than 1.44 %. Multiplying Eqs (17) and (15) gives the corresponding upper bound on the ratio of tube power in a finite-diameter tube to vehicle power in an infinite-diameter tube as

$$P_t / P_{v\infty} \leq (P_t / P_v)(P_v / P_{v\infty}) = (0.0144)(1.34) = 0.019 \tag{18}$$

which is nearly 2 %. Although tube power is small, it could be – depending on how close the upper bound is to the true value – large enough not to be ignored.

We can derive an approximation to the right-hand side of Eq. (16) that is easier to compute than numerical integration. A trainset is cylindrical over most of its length: The four-passenger-car trainset considered above has a total length M = 106.6 m, but the length of the two nose-cones is $2L$ = 13.4 m, and thus the trainset is cylindrical for 87 % of its length. Hence, the gap can be approximated by the gap formed between the tube inner surface and a vehicle cylindrical surface over its entire length. With this assumption, the velocity mean-value function in the numerator of Eq. (16) is approximated by the constant $\Delta V(x^*)$ and the velocity mean-value function in the denominator is approximated by the constant $V_g(x^*)$, where x^* is a maximum point of function δ_v. Observing that, in this case, factor $S_t / S_v = q(x^*)$, namely, the fixed ratio $\delta_t / \delta_v(x^*)$ of tube diameter to vehicle maximum diameter, we have

$$P_t / P_v \leq (S_t / S_v)[\int_0^M \Delta V(x)^2 \, dx \, M^{-1} / \int_0^L V_g(x) \, dx \, L^{-1}]^3 \cong q(x^*)^{-5} \tag{19}$$

as an approximation of the upper bound given by Eq. (16). Eq. (19) estimates the upper bound as 0.013, which is about 7 % lower than the value 0.0144 given by numerical integration in Eq. (17).

IV. Results and Discussion

Although our interest is aerodynamics in a hydrogen atmosphere, the results for gap flow and gap drag apply to any perfect gas (e.g., air) that satisfies the principal assumptions of one-dimensional, inviscid, incompressible flow.

This analysis of gap flow allows determination of a fuselage shape and tube diameter that optimize design parameters of the supersonic tube vehicle: While fixing the maximum vehicle diameter at $\delta_v(x^*) = 2.69$ m, we desire simultaneously to (a) limit gas velocity through the propeller to Mach 0.74 and (b) limit gas velocity over the vehicle surface to Mach 0.90. A fuselage diameter of 2.69 m is the diameter of the Bombardier Q400 turboprop airplane, which served as the model for aerodynamic extrapolation of the power of the supersonic tube vehicle.[1] The previous paper in this series[2] showed that, given a sufficiently large tube, avoidance of shocks at the propeller blade-tips limits vehicle speed to V_∞ = Mach 0.74. Although Mach 0.90 is within the usual transonic region, it is a reasonable velocity over the featureless outer cylindrical surface that has no curvature along its length (after the transition from the nose); turbofan airplanes, with less favorable features, cruise at speeds as high as Mach 0.855.

Table 1. Gas velocity over vehicle surface for tube diameter δ_t = 6.4 m

Distance x m	Diameter $\delta_v(x)$ m	Velocity $V_g(x)$ Ma
0	0	0.740
1.20	0.500	0.744
2.40	1.33	0.773
3.60	2.03	0.823
4.80	2.46	0.868
6.00	2.66	0.895
7.20	2.69	0.899

Limiting flow velocity through the propeller led to the bottle-nose design shown in Figures 1 and 2. The bottle-nose maximizes $q(x)$ in the propeller region. Using an iterative search procedure, Eq. (6) was used to calculate a tube diameter of δ_t = 6.4 m that satisfies the design constraints (a) and (b) above. Table 1 shows a subset of surface flow velocity data for the optimized case. The table displays evenly spaced values of x, the distance in meters from the tip of the vehicle (x = 0) along its centerline. Diameter $\delta_v(x)$ is the diameter of the vehicle at each value of x. Given the profile data $\delta_v(x)$, tube diameter δ_t = 6.4 m, and V_∞ = 0.74 Ma, Eq. (6) calculates $V_g(x)$ as the third column of the table.

Using Eq. (9), with the maximum flow velocity in Table 1, namely, $V_g(x^*) = 0.90$ Ma, δ_t = 6.4 m, and $\delta_v(x^*)$ = 2.69 m at maximum point x^*, I calculate the maximum flow velocity through the gap as $\Delta V(x^*) = 0.16$ Ma. The assumption of incompressibility is consistent with this low subsonic flow velocity, the highest velocity in the tube aside from flow immediate to the vehicle surface.

The results provide an estimate of vehicle parasitic power due to gap flow and consequent increased drag on the vehicle surface and tube drag. Equations (14) and (15) indicate that gap flow increases vehicle drag by 22 % and required vehicle power by 34 %. The ratios will approach unity as closely as desired by increasing the tube diameter; however, the larger the tube, the higher the infrastructure cost of a tube transportation system. When tube drag is included, Equation (17) shows that the parasitic drag increases additionally by less than 2 %. Together, the sources of gap drag increase required vehicle power by at most (1.34) (1.0144) = 1.36, or 36 %. Because gap drag depends only on the ratio of tube diameter to vehicle diameter, this result holds for any vehicle and tube system for which the ratio of tube-to-vehicle diameter is 2.38.

In the original power calculation,[1] the power required to propel the supersonic tube vehicle at Mach 1.2, exclusive of gap drag, was calculated as 1.1 MW. Multiplying this by the factor 1.36 just computed gives 1.50 MW as an upper bound on vehicle power when gap drag is included.

Conclusions

Gap flow, accelerated gas flow through the gap formed by the vehicle outer surface and tube inner surface, is a function only of the ratio of tube diameter to vehicle diameter. The results for gap flow and gap drag apply to any perfect gas that satisfies the principal assumptions of one-dimensional, inviscid, incompressible flow in the gap. The equations derived for gap flow, gap drag, and gap power – Equations (6), (8), (9), (12), (13), and (16) – have led to

(a) calculation of a tube diameter (6.4 m) that is consistent with design constraints, (b) introduction of the design feature of the bottle-nose to minimize shocks at the propeller blade tips due to gap flow, and (c) a better upper bound on the power required to propel the supersonic tube vehicle at supersonic speed with respect to air outside the tube. Gap flow increases the required power by at most 36 % for any vehicle and tube system for which the ratio of tube-to-vehicle diameter is 2.38.

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